Sequential Decision Model for the Single Airport Ground Holding Problem

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Outline

- Introduction and background
- Model and formulation
- Computational strategy
- Concluding remarks

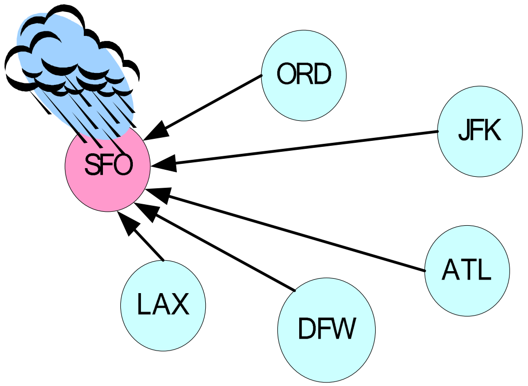
Ground Holding Problem

- Ground Delay Program
  - Mitigate destination airport capacity-demand imbalances by assigning delays to flights at their origin airports

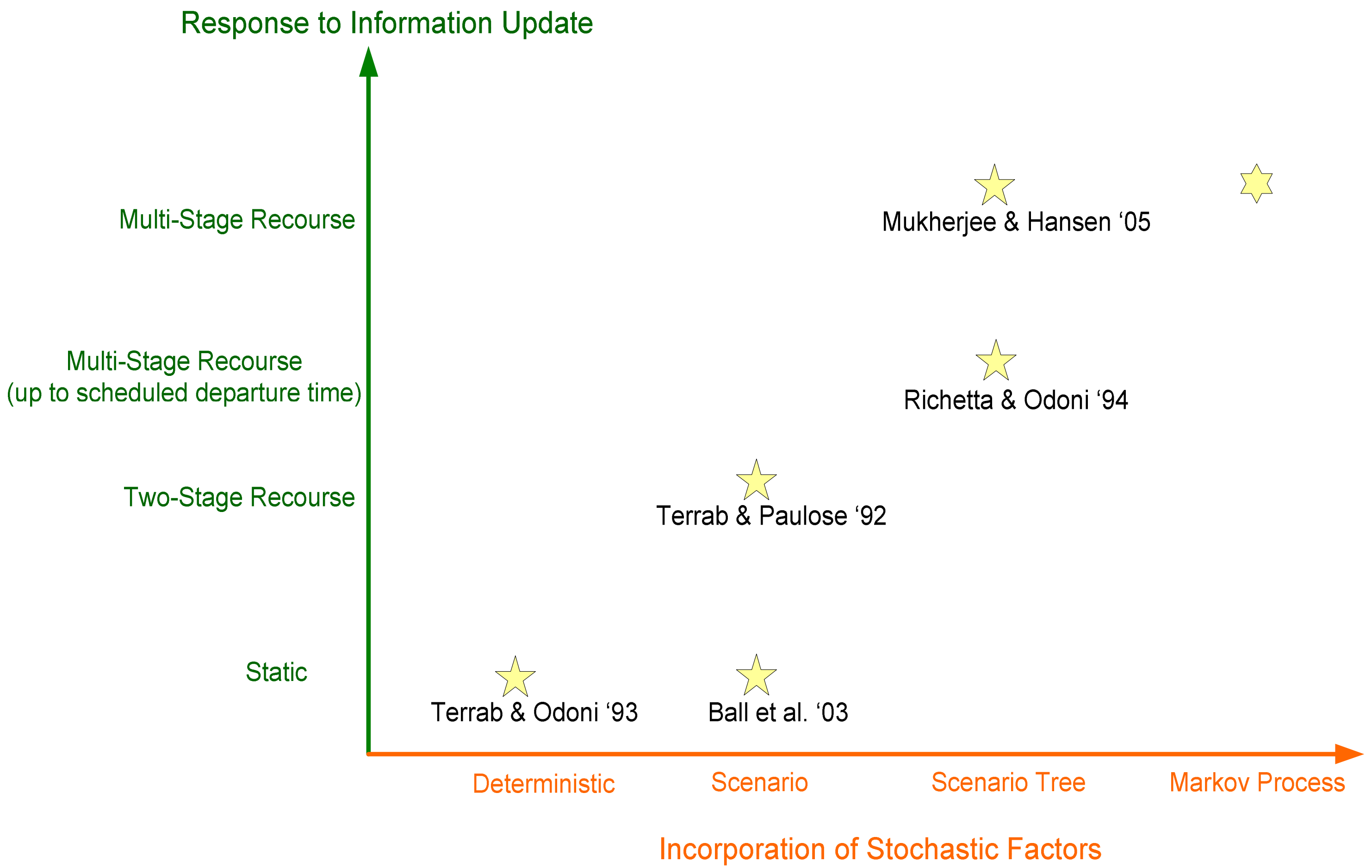
- Ground Holding Problem (GHP)
  - Rationale: airborne delay is more expensive than ground delay
  - Approach: assign ground delays to flights
  - Goal: minimize Expected Total Delay Cost

  → **`minimize {ground delay cost + E[airborne delay cost]}`**

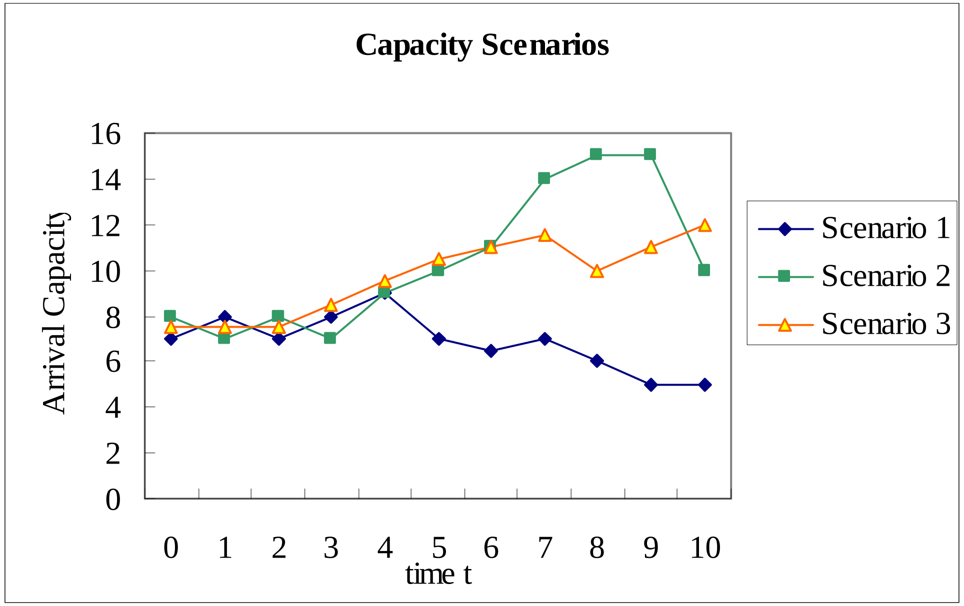
- Single Airport Ground Holding Problem (SAGHP)
  - SAGHP solves the GHP for one destination airport

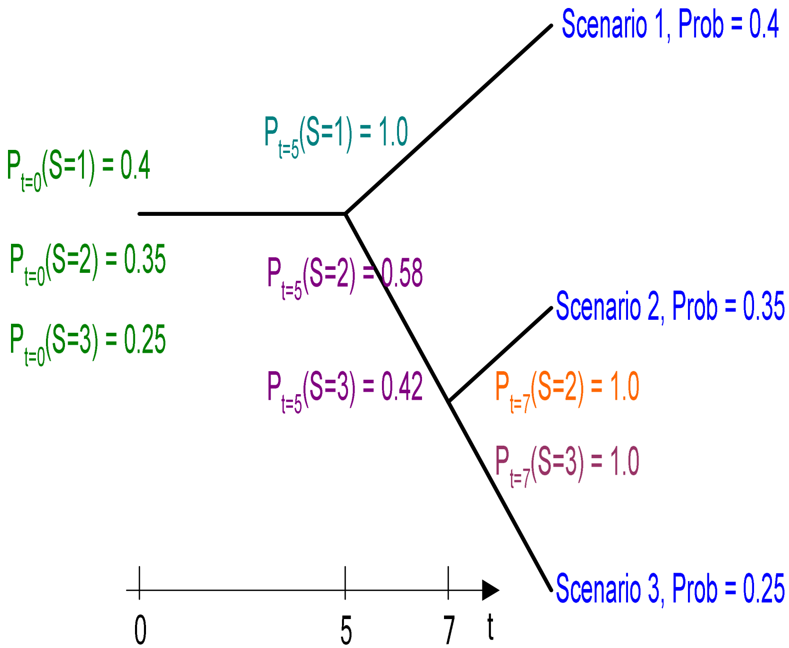


Evolution of Models for the SAGHP



Capacity Scenarios and Scenario Tree

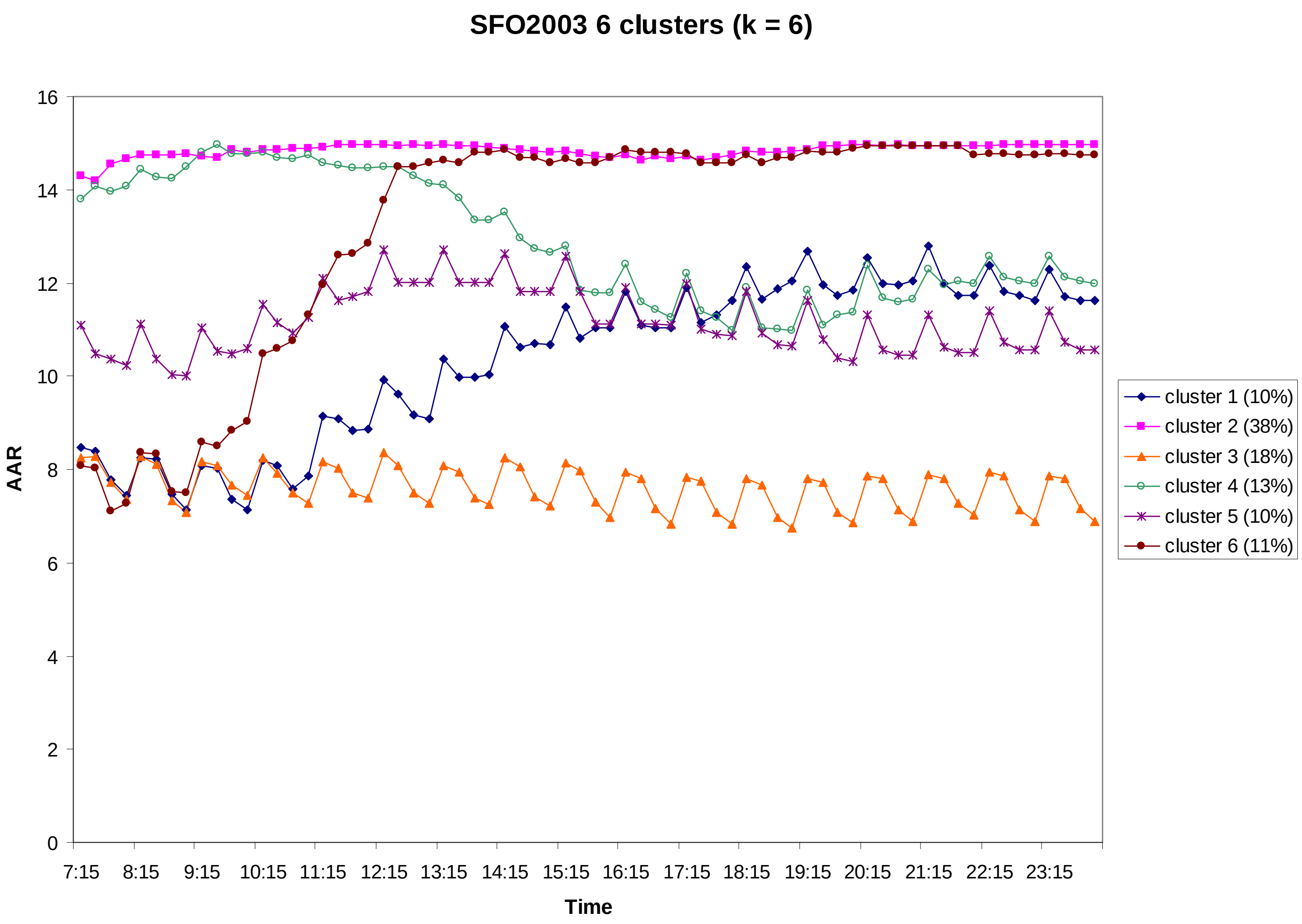




Use of capacity scenarios and scenario trees in optimization models

- Static Model—capacity constraint
- Dynamic Model—capacity constraint and information update

Sample Capacity Scenarios at SFO



Shortcomings of Scenario-based Methods

- Fundamental problems
  - Impose a finite-scenario tree structure on a reality where there is a much larger set of possibilities for capacity evolution
  - Not to utilize improved information about future capacity which can be obtained continually rather than at a few discrete branching points
- Issues found in empirical studies
  - Costs incurred from applying the output of scenario-based optimization models is considerably higher than the theoretical optimization results
    - The actual capacities vary around the nominal values assumed in the optimization
    - Uncertainty in correctly identifying the scenario that matches best with the condition

Research Goal

- Improve the ability of air traffic managers to handle uncertainty and incorporate probabilistic forecast information in ground delay programs

- Learn from the shortcomings of the scenario-based models for SAGHP and explore scenario-free alternatives
  - Propose a scenario-free sequential decision model for the SAGHP
  - Develop computational strategies and demonstrate computational feasibility

Model and Formulation

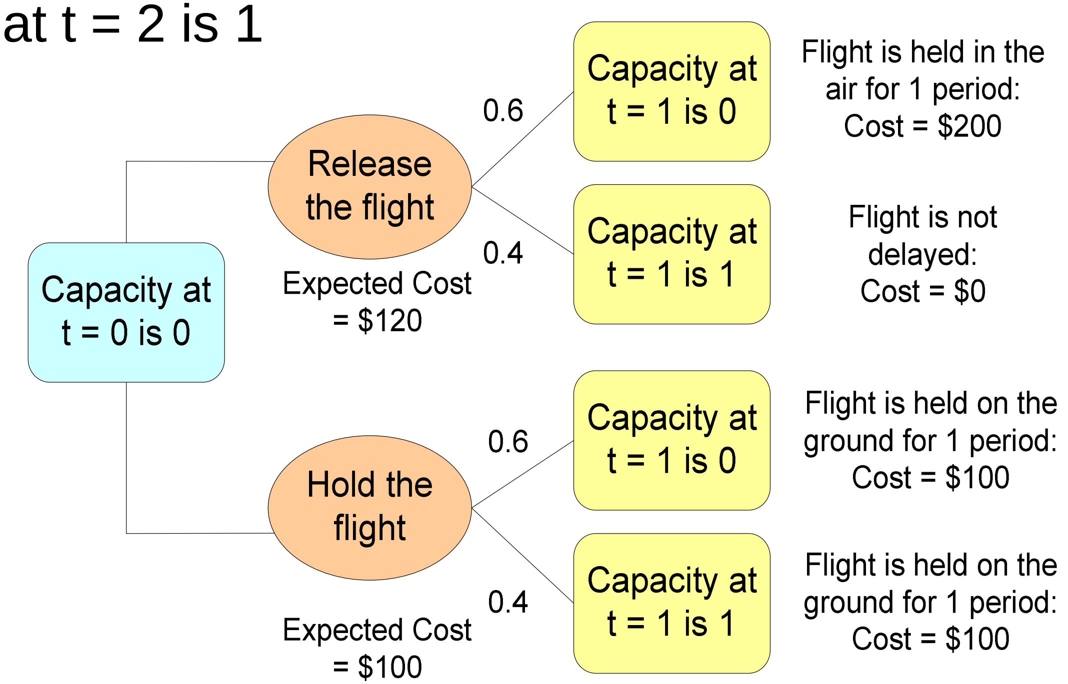
- Markovian capacity evolution process
- Sequential decision model
- Dynamic programming formulation
- Algorithmic complexity

Modeling using a Markovian Approach
- An Example

- Suppose capacity evolution is Markovian with transition matrix:

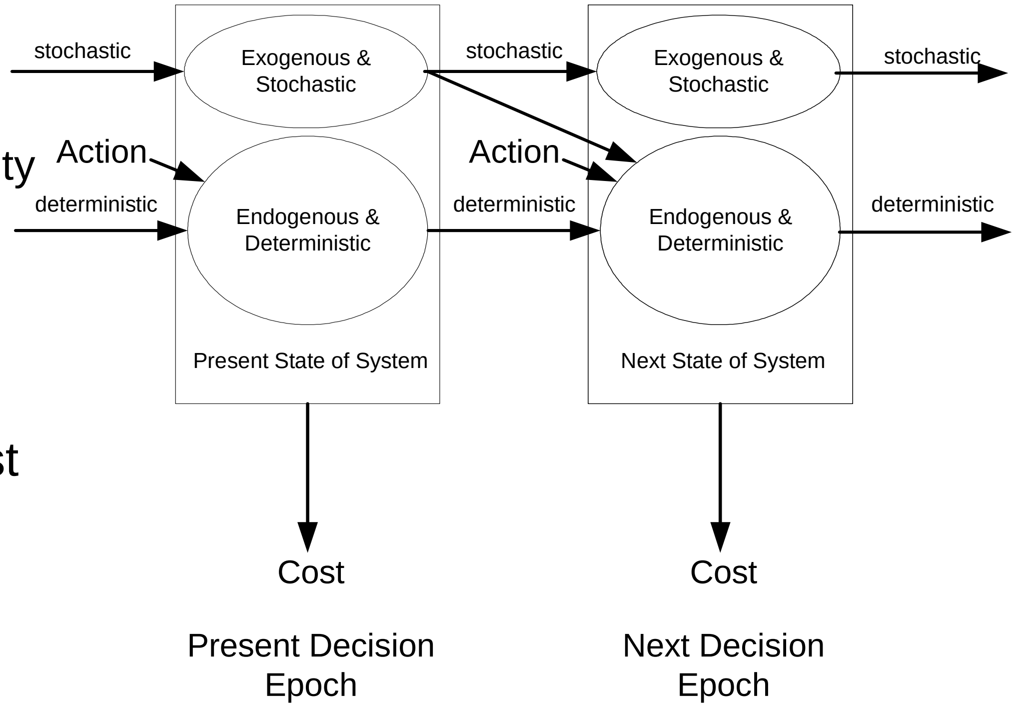
$$\begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$$

- Flight time: 1 period
- Capacity at t = 2 is 1



Sequential Decision Model for the SAGHP

- Decision epoch
  - every quarter hour
- State
  - destination airport arrival capacity
- Available actions
  - number of flights to hold
- State and action dependent cost
  - ground and airborne delay cost
- State dependent transition probabilities
  - arrival capacity transition probabilities



Dynamic Programming Formulation

- Optimality Equation

$$f_t(K_t) = \min_{H_t \le G_t + D_t} \left\{ c_g H_t + c_a W_t + \mathrm{E}\left[f_{t+1}(K_{t+1}) \right] \right\} \qquad \text{where } \begin{array}{l} G_{t+1} = H_t, G_0 = D_0, G_T = 0 \\ W_t = (L_t - K_t)^+ \end{array}$$

$$= \min_{H_t \le G_t + D_t} \left\{ c_g H_t + c_a W_t + \sum_{K_{t+1} = K_{\min}(t+1)}^{K_{\max}(t+1)} P_{K_t K_{t+1}} \cdot f_{t+1}(K_{t+1}) \right\}$$

c_g = cost of ground delay for one time period per flight.
c_a = cost of airborne delay for one time period per flight.
G_t = number of flights queued on ground by time t.
D_t = number of flights scheduled for departure at time t.
H_t = number of flights to hold on ground at time t.
L_t = number of flights ready to land at time t.
W_t = number of flights experiencing airborne delay at time t.
K_t = arrival capacity at the airport at time t.
$K_{max}(t)$ = maximum arrival capacity at the airport at time t.
$K_{min}(t)$ = maximum arrival capacity at the airport at time t.
$P_{kk'}$ = transition probability from arrival capacity k to k' in the next period.

Formulation—flight specific holding

- Optimality equation

$$f_t(K_t) = \min_{X_f^t,\, f\in F} \left\{ c_g \sum_{f\in F} (X_f^t - S_f^t) + c_a W_t + \sum_{K_{t+1}=K_{\min}(t+1)}^{K_{\max}(t+1)} P_{K_t K_{t+1}} \cdot f_{t+1}(K_{t+1}) \right\}$$

s.t.

$$Y_f^{t+1+\tau_f} = X_f^t - X_f^{t+1} \qquad \forall f \in F,\, t = 0,\ldots,T-1$$

$$A_t = \sum_{f\in F} Y_f^t \qquad t = 0,\ldots,T$$

$$W_t = (L_t - K_t)^+ \qquad t = 1,\ldots,T; \quad W_0 = 0$$

$$L_t = A_t + W_{t-1} \qquad t = 1,\ldots,T$$

$$X_f^t \ge S_f^t \qquad \forall f \in F,\, t = 0,\ldots,T$$

$$X_f^t \ge X_f^{t+1} \qquad \forall f \in F,\, t = 0,\ldots,T-1$$

$$X_f^t \in \{0,1\}, \quad Y_f^t \in \{0,1\} \qquad \forall f \in F,\, t = 0,\ldots,T$$

where

$$X_f^t = \begin{cases} 1 & \text{if flight } f \text{ stays on the ground during time period } t \\ 0 & \text{otherwise} \end{cases} \qquad \forall f, \forall t.$$

$$Y_f^t = \begin{cases} 1 & \text{if flight } f \text{ is planned by the model to arrive in time period } t \\ 0 & \text{otherwise} \end{cases} \qquad \forall f, \forall t.$$

$$S_f^t = \begin{cases} 0 & \text{if flight } f \text{ is scheduled to depart in or before time period } t \\ 1 & \text{otherwise} \end{cases} \qquad \forall f, \forall t.$$

τ_f = the duration of flight of flight *f*.

A_t= the number of flights planned by the model to arrive in time period *t*.

Formulation—duration group-based holding

- Optimality equation

$$f_t(K_t) = \min_{\substack{0 \le Z_\gamma^t \le G_\gamma^t + S_\gamma^t \\ \gamma \in \Gamma}} \left\{ c_g \sum_{\gamma \in \Gamma} Z_\gamma^t + c_a W_t + \sum_{K_{t+1}=K_{\min}(t+1)}^{K_{\max}(t+1)} P_{K_t K_{t+1}} \cdot f_{t+1}(K_{t+1}) \right\}$$

s.t.

$$G_\gamma^t = Z_\gamma^{t-1} \qquad \forall \gamma \in \Gamma, \quad t = 1, \ldots, T; \quad G_\gamma^0 = 0, \forall \gamma \in \Gamma$$

$$\xi_\gamma^{t+\tau_\gamma} = G_\gamma^t + S_\gamma^t - Z_\gamma^t \qquad \forall \gamma \in \Gamma, \quad t = 0, \ldots, T-1$$

$$A_t = \sum_{\gamma \in \Gamma} \xi_\gamma^t \qquad t = 1, \ldots, T$$

$$W_t = (L_t - K_t)^+ \qquad t = 1, \ldots, T; \quad W_0 = 0$$

$$L_t = A_t + W_{t-1} \qquad t = 1, \ldots, T$$

$$Z_\gamma^t \in Z^+ \qquad \forall \gamma \in \Gamma, \quad t = 0, \ldots, T-1$$

where Γ = the set of groups that represent the classification of flights.
Z_γ^t = number of flights of group γ to hold on the ground in time period *t*.
ξ_γ^t = number of group γ flights to arrive in time period *t*.
S_γ^t = number of group γ flights scheduled for departure at time *t*.
G_γ^t = number of group γ flights queued on ground from previous periods by time *t*.

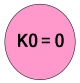
A Simple Example

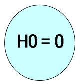
Example:

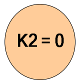
# of Decision Stages: 2

Capacity Levels: 0 and 1

Transition matrix: $\begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} \end{array}$

Decision Node: eg. 

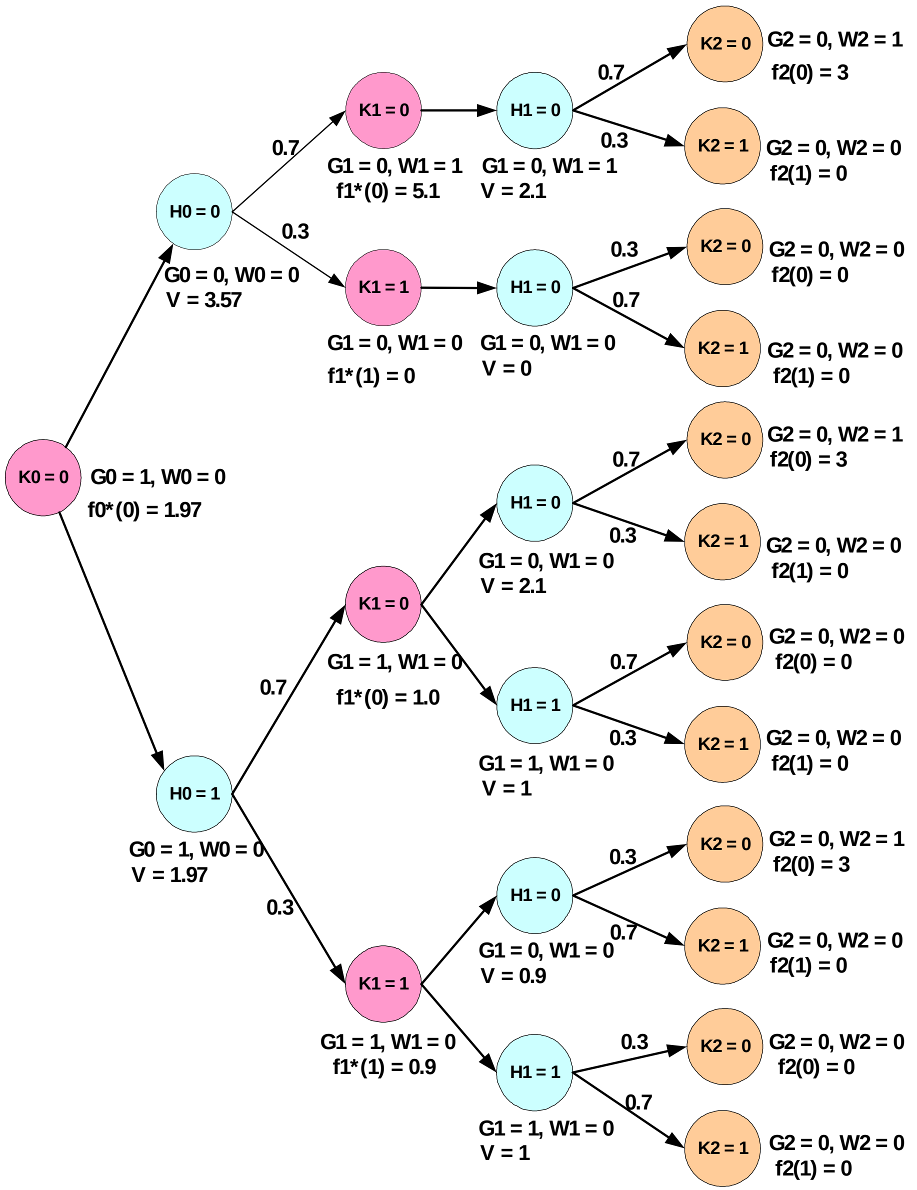
Action Node: eg. 

Terminal Node: eg. 

in queue on ground: Gt

in queue in the air: Wt

Cg = 1, Ca = 3



Algorithmic Complexity

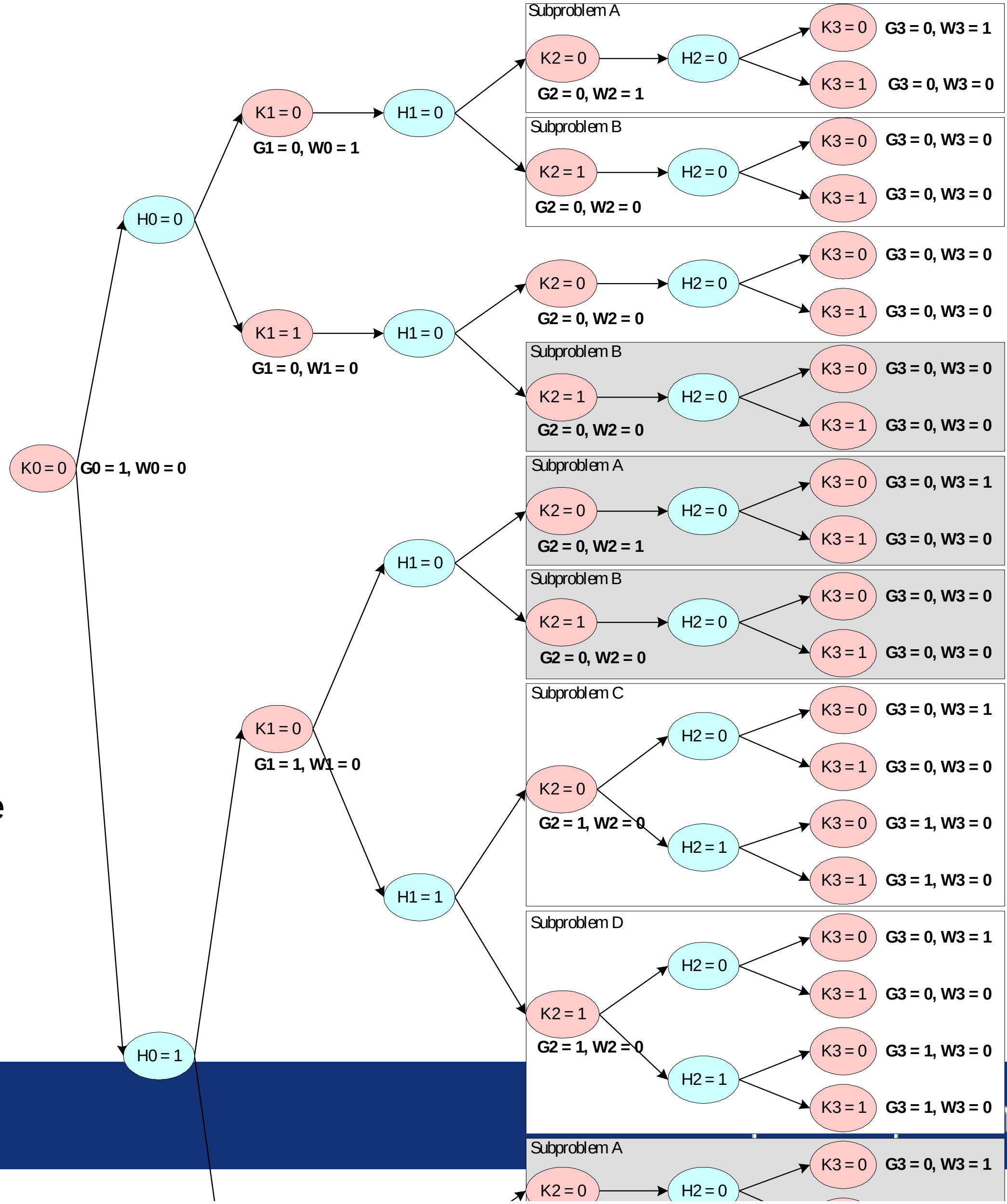
- Value iteration algorithm
  - Policy improvement (decision node)
  - Policy evaluation (action node)
- Complexity for duration group-based holding is $O(((\frac{F}{G})^G N)^T)$
  - F = the number of flights to release in a decision stage
  - G = the number of groups of flights
  - T = the number of decision epochs in the planning horizon
- If there exists a priority ordering among the flights such that the grouping is not needed, the complexity will be $O((FN)^T)$

Computational Strategies

- Memoization
- Priority ordering
- Limited Search

Memoization—identify the overlapping subproblems

- Quintuple (t, K_t, B_t, V_t, W_t) is necessary and sufficient to characterize a subproblem
  - K_t: arrival capacity at t
  - B_t: the bag of flight durations for the flights ready to be released in t
  - A_t: the number of flights released and planned to arrived in t
  - V_t: the vector of A_t's after t $<A_{t+1}, A_{t+2},\ldots, A_T>$
  - W_t: the length of the airborne queue at t



Priority Ordering

- Reduce the time complexity from $O(((\frac{F}{G})^G N)^T)$ to $O((FN)^T)$
- Priority ordering schemes:
  - Longest Goes First (LGF): flight with longer duration has the priority
  - Ration by Schedule (RBS): flight with earlier scheduled arrival time has the priority

Limited Search—
Structural property of the cost-to-go function

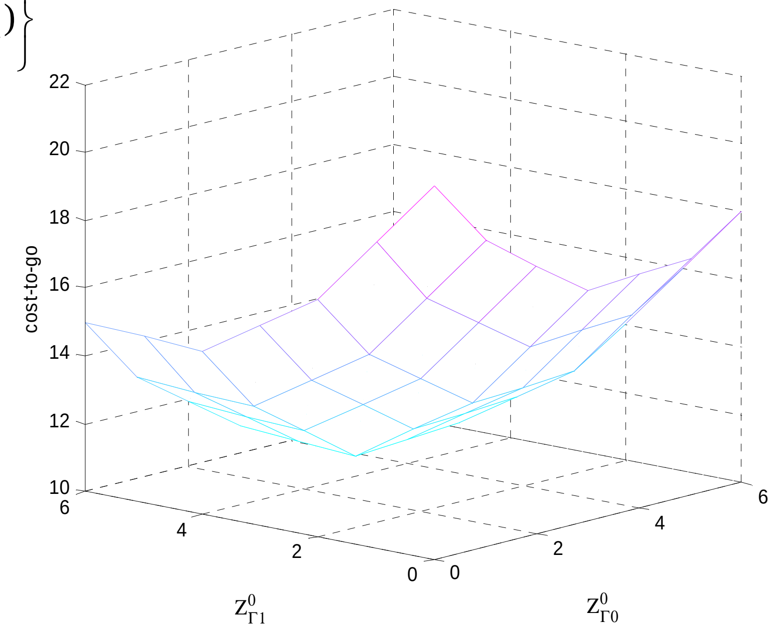
- The cost-to-go function is convex in the number of flights to hold

$$f_t(K_t) = \min_{\substack{0 \le Z_\gamma^t \le G_\gamma^t + S_\gamma^t \\ \gamma \in \Gamma}} \left\{ c_g \sum_{\gamma \in \Gamma} Z_\gamma^t + c_a W_t + \sum_{K_{t+1} = K_{\min}(t+1)}^{K_{\max}(t+1)} P_{K_t K_{t+1}} \cdot f_{t+1}(K_{t+1}) \right\}$$

  is convex in $Z_\gamma^t, \forall \gamma \in \Gamma$, where Γ is the set of duration-based groups of flights

  - Non-negative integer decision variables (# of flights to hold)
  - Piece-wise linear function
  - Discrete & non-differentiable
    - → Combinatorial optimization

- When there is only one group?



Cost-to-go function with two groups of flights

Limited Search—
cost-to-go with priority ordering

- Example: construction of cost-to-go function

$$f(x) = c_g x + p_1 c_a (2 - x)^+ + p_2 c_a (4 - x)^+$$

$$y^+ \equiv \max(0, y)$$

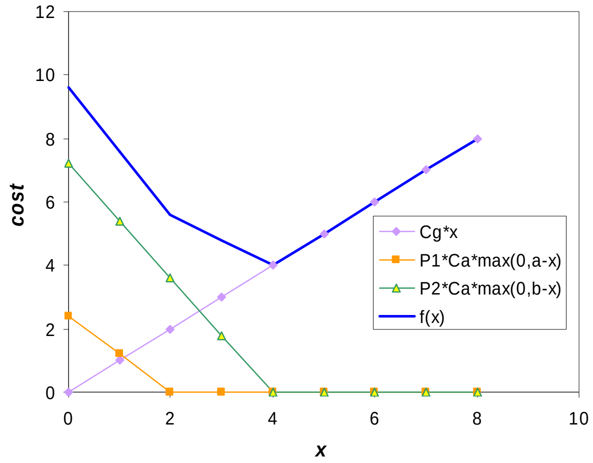
- Example: cost-to-go function when priority ordering is adopted

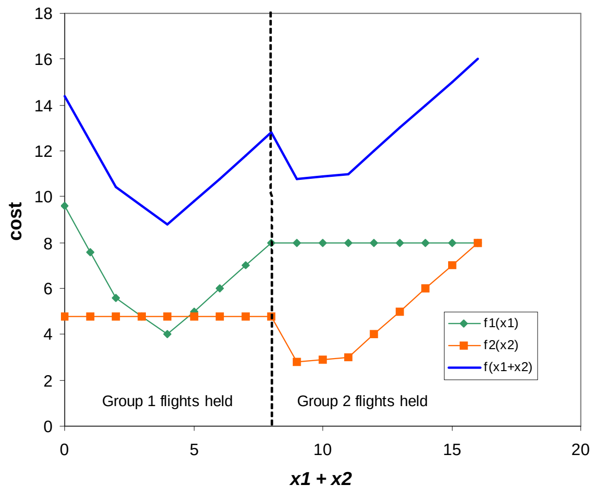
$$f(x) = f(x_1 + x_2) = f_1(x_1) + f_2(x_2)$$

$$f_1(x_1) = c_g x_1 + 0.4 \cdot c_a (2 - x_1)^+ + 0.6 \cdot c_a (4 - x_1)^+$$

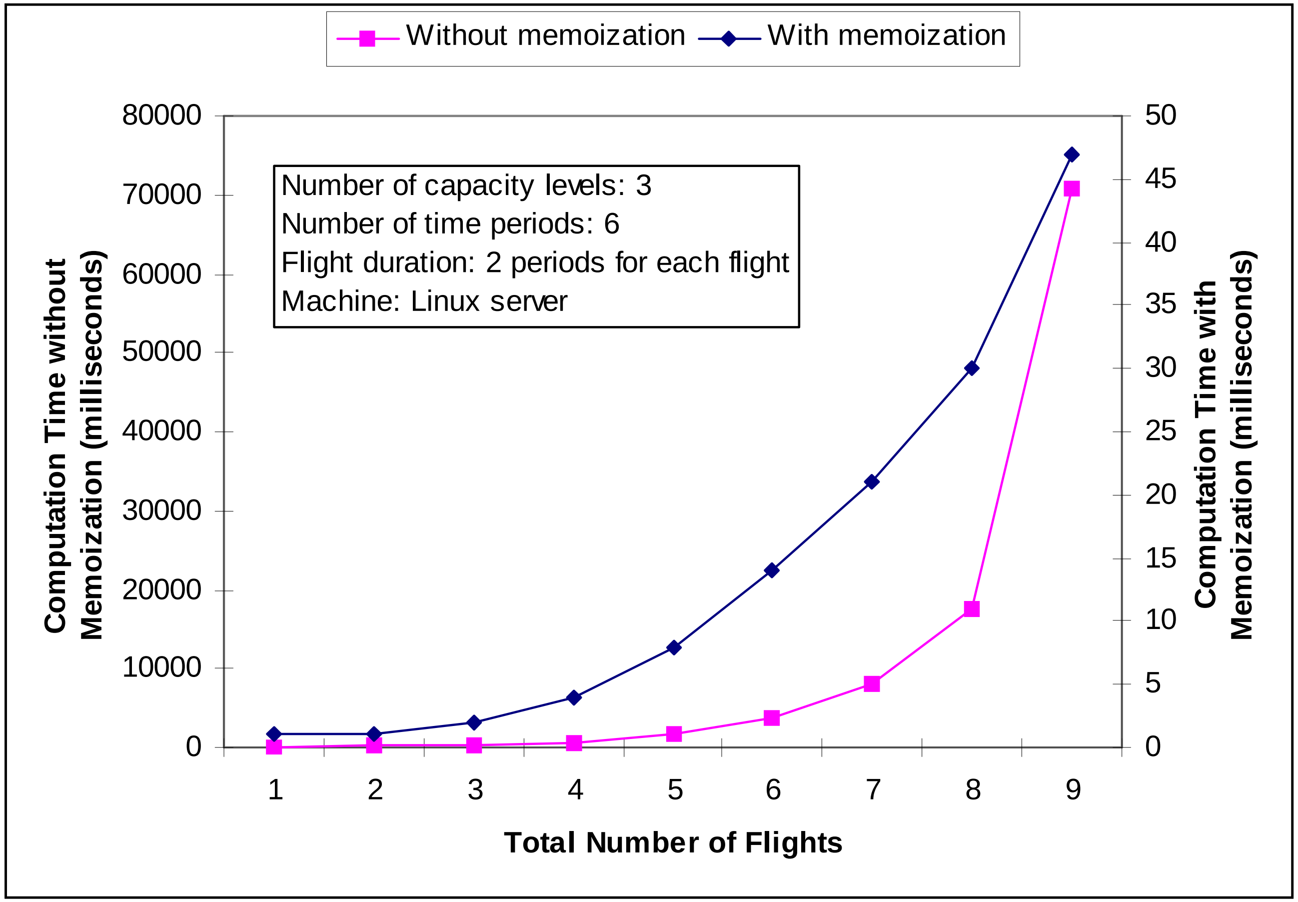
$$f_2(x_2) = c_g x_2 + 0.7 \cdot c_a (1 - x_2)^+ + 0.3 \cdot c_a (3 - x_2)^+$$

- Limited search heuristics
  - H1: Assume convexity, search from both ends
  - H2: Assume convexity, search from the lower end

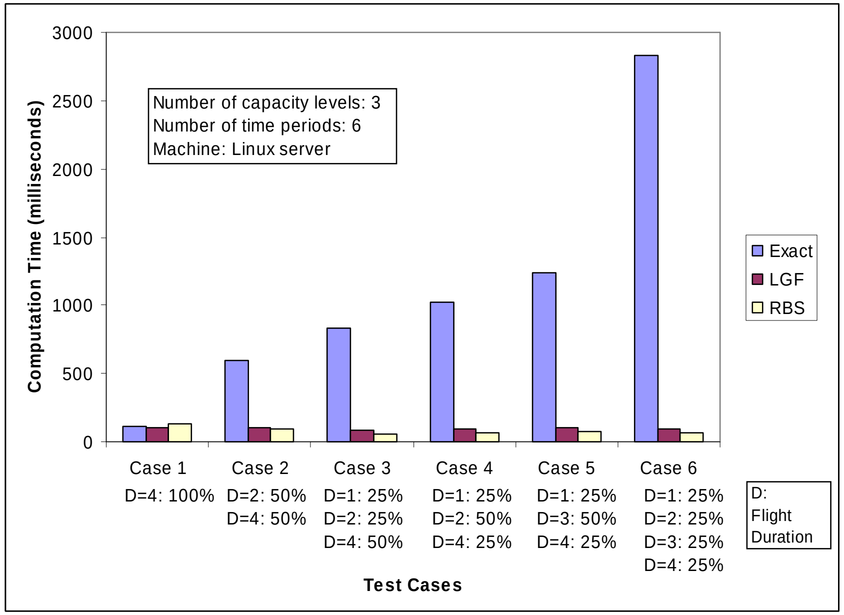


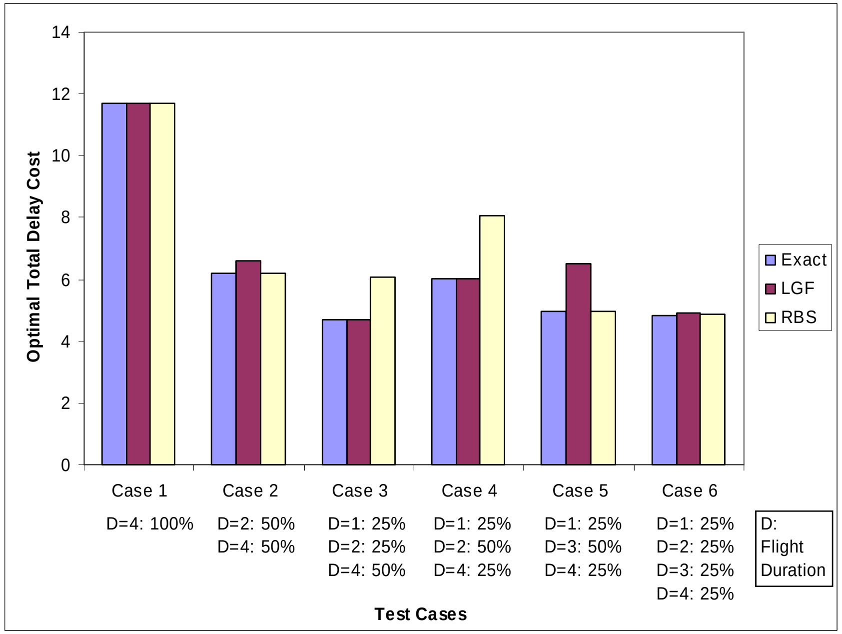


Computational Result: Effect of Memoization

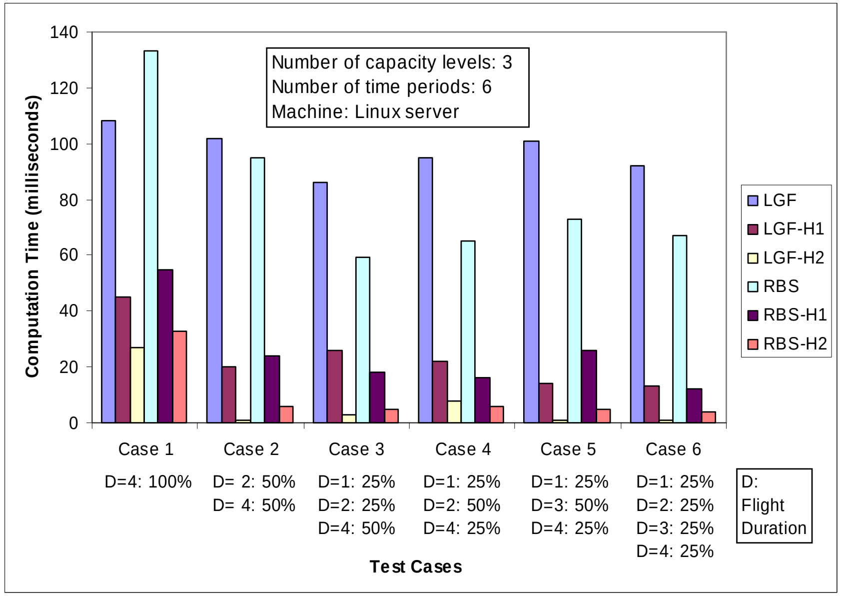


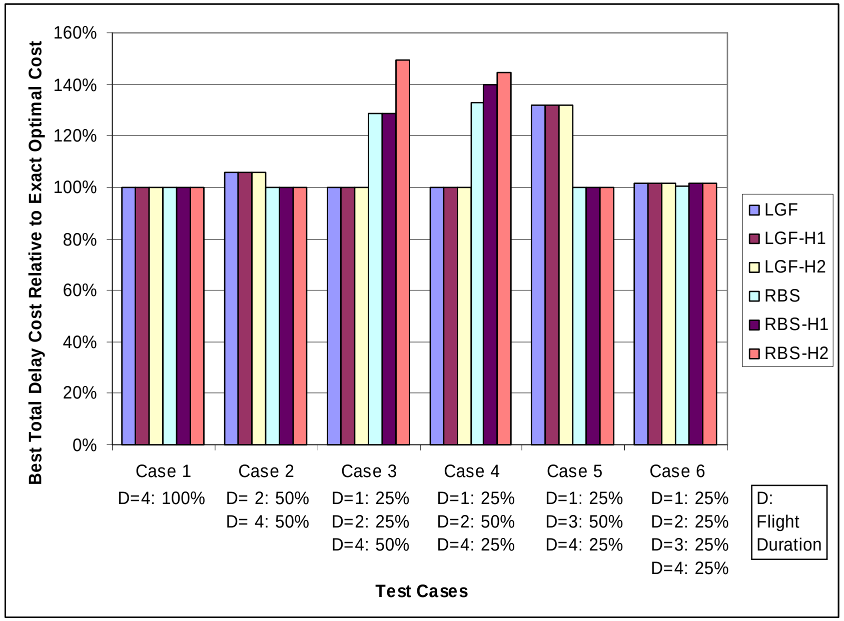
Computational Result:
Effect of Priority Ordering





Computational Result:
Effect of Limited Search





Scenario-free Model with Real World Data

- Date: March 2nd, 2006
- Location: SFO
- Transition matrix: 3 x 3
- Planning horizon: 7am to 2pm
- Flights affected: 116 flights departing between 7am and 12 noon
- Result:

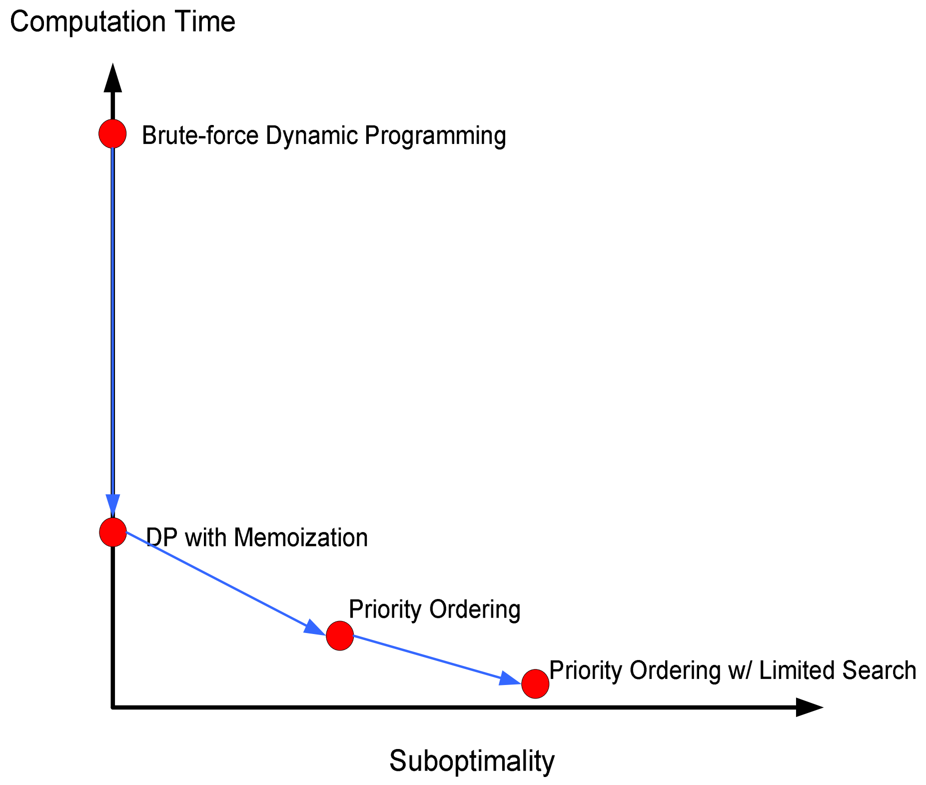
	Priority Ordering	
	LGF	**RBS**
Number of Decision Nodes	156301	415228
Optimal Expected Total Delay Cost	26.43	25.87
Computation Time (milliseconds)	2324	19741

Summary

- The performance of a scenario-based model is compromised by a few shortcomings

- Development of scenario-free sequential decision model for SAGHP
  - Dynamic programming formulation
  - Computational strategies
    - Overlapping subproblems
      → Memoization
    - Complexity reduction
      → Priority ordering
    - Structural property
      → Limited search

- Computationally feasible for problems of real-world scale

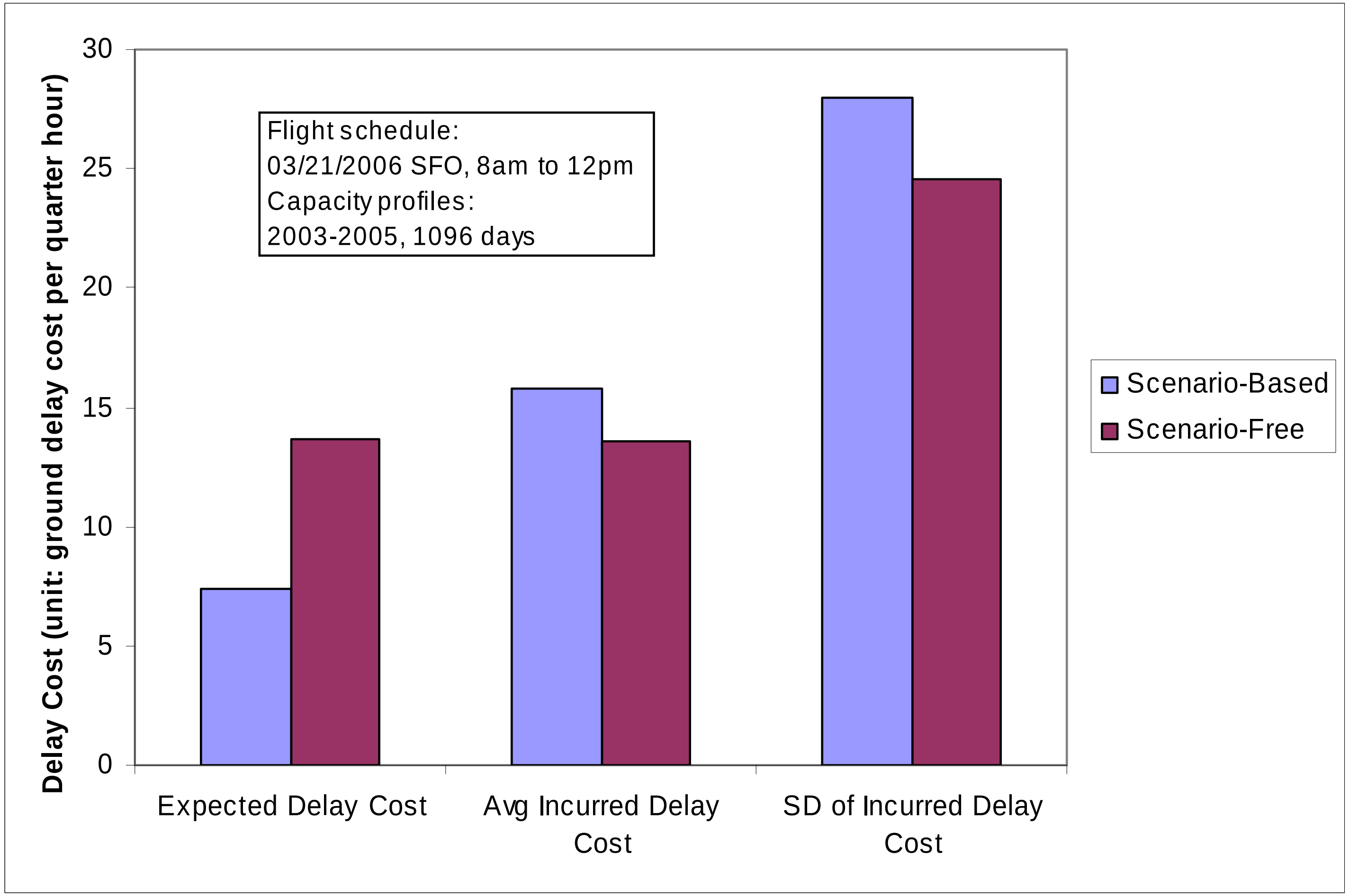


Latest Development

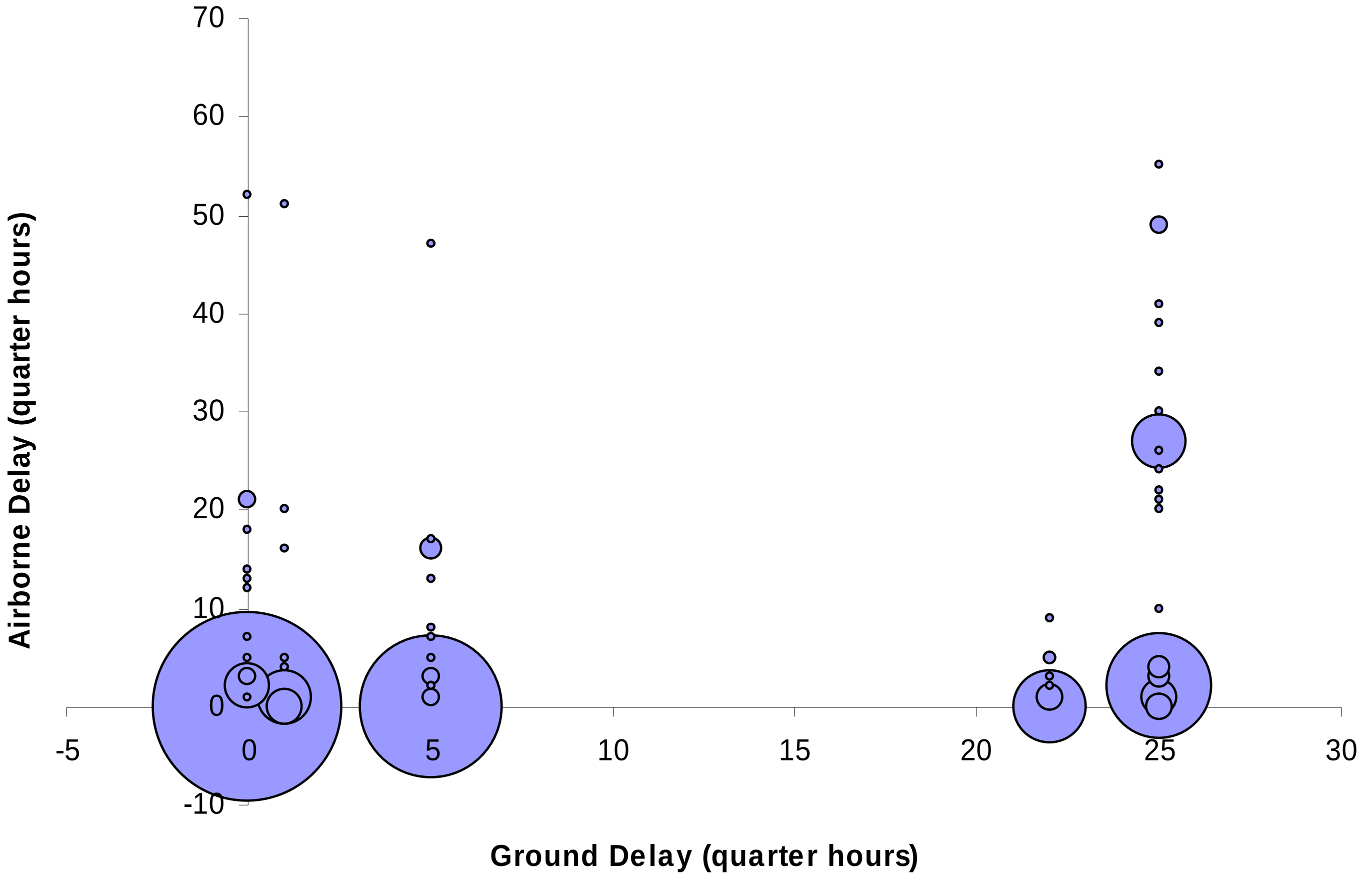
- Comparison of scenario-based and scenario-free model
  - Theoretical comparison: equivalence and tradeoffs
  - Scenario-free model led to lower average incurred delay cost and lower variation in costs
  - Scenario-free model led to closer expected and incurred cost
  - Scenario-free model yields solutions that contain more balanced distribution of ground and airborne delay

- Future Work
  - Methodology for transition matrix estimation
  - Estimator of the computational cost
  - Other strategies to manage the complexity
  - Generalizability of the performance results
  - Accommodate different risk preferences using the scenario-free approach
  - Incorporation of the scenario-free approach into CDM

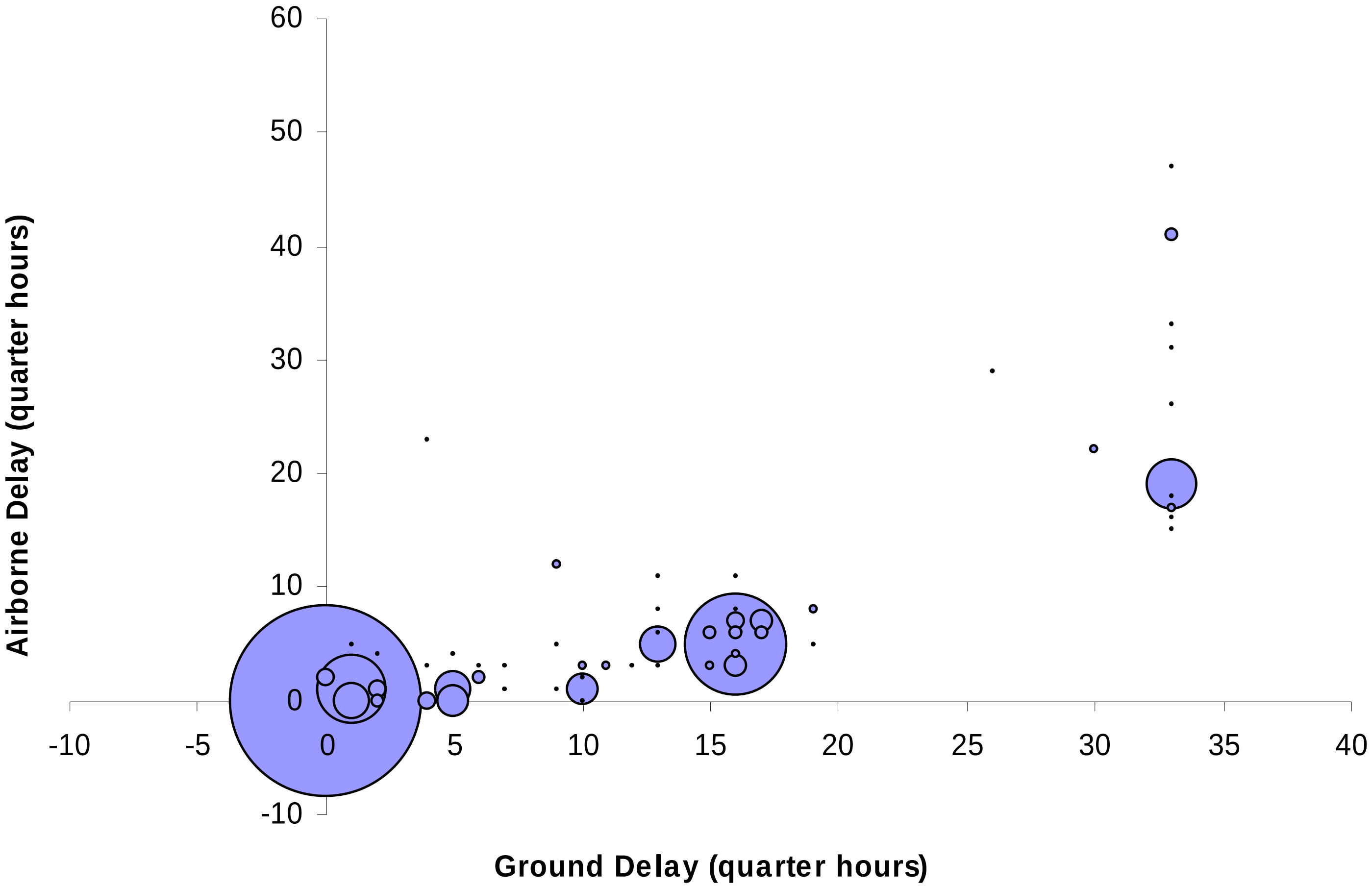
Lower Average Incurred Delay Cost
Lower Variation in Costs
Closer Expected and Incurred Delay Cost



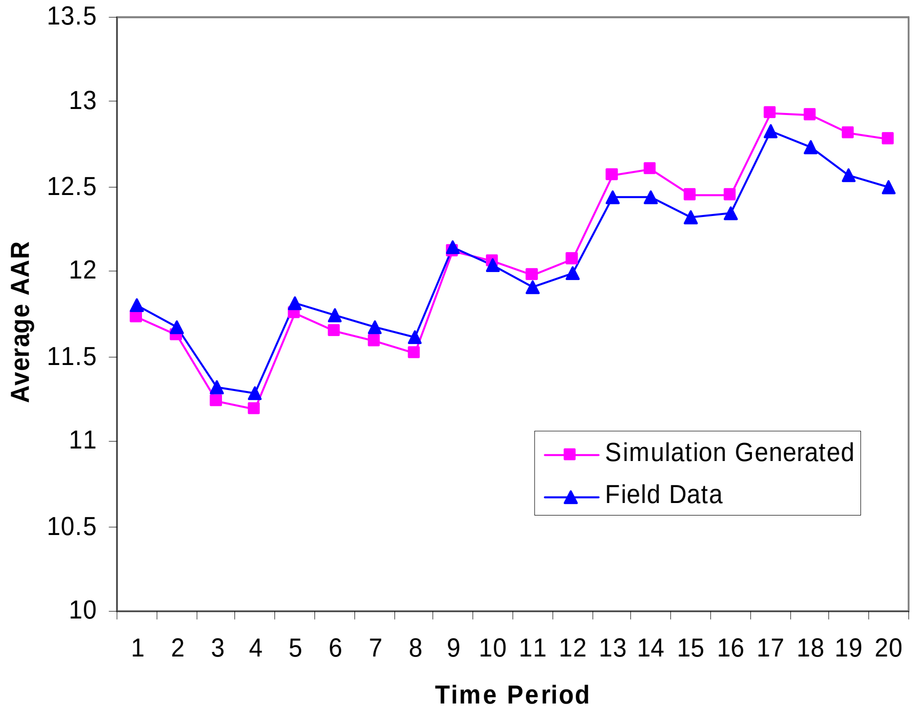
Delay Distribution: Scenario-Based Model

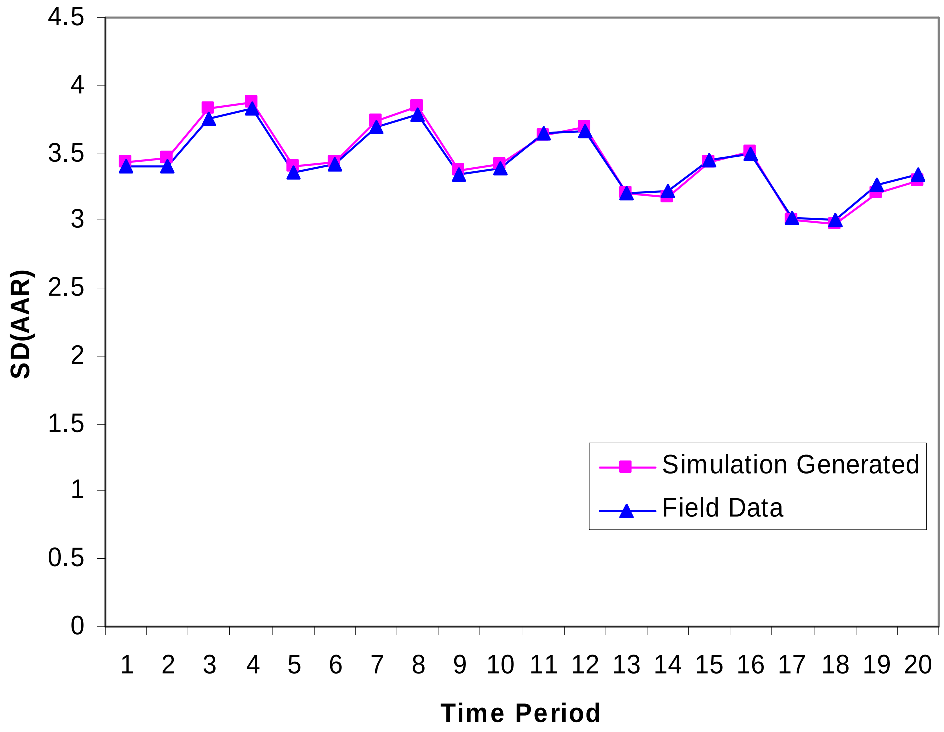


Delay Distribution
Scenario-Free Model

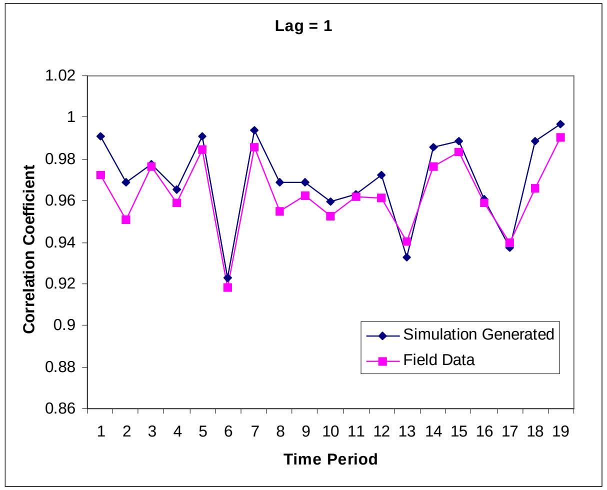


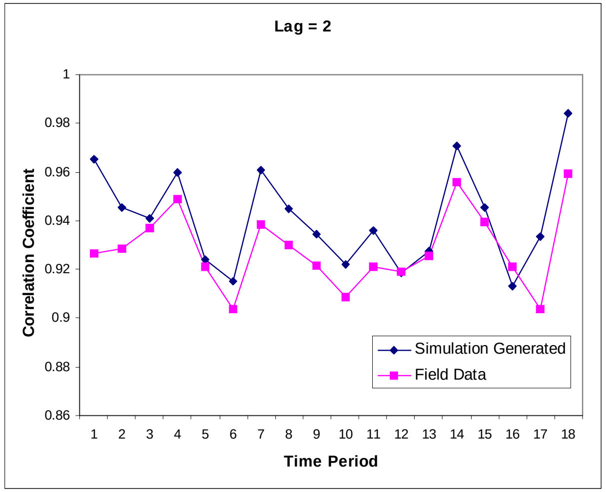
Realism of the Markovian Model (I)

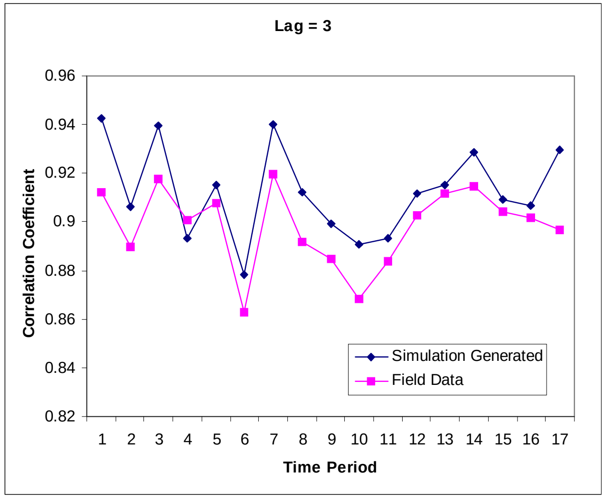


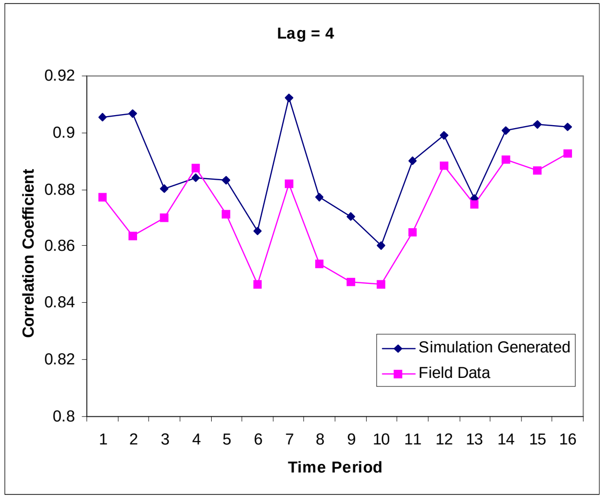


Realism of the Markovian Model (II)









Description of Test Cases

- Flight schedule for the test cases

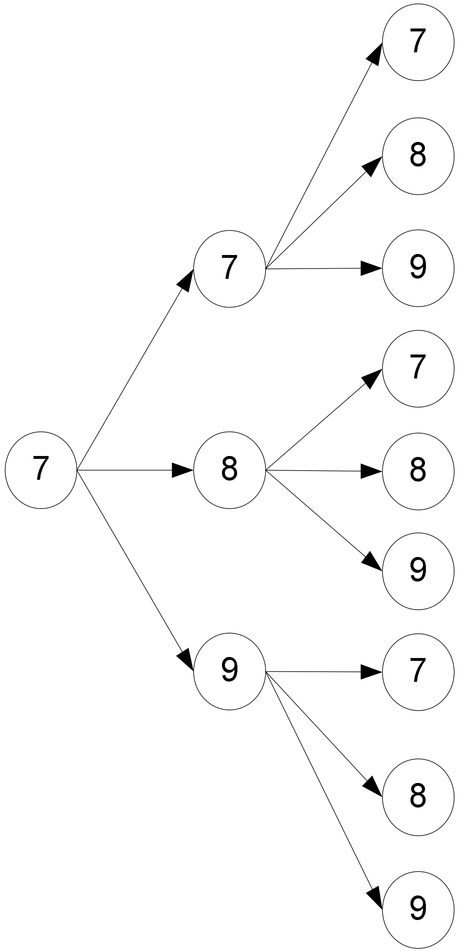
Flight Index	Case 1		Case 2		Case 3		Case 4		Case 5		Case 6	
	Departure Period	Arrival Period	Departure Period	Arrival Period	Departure Period	Arrival Period	Departure Period	Arrival Period	Departure Period	Arrival Period	Departure Period	Arrival Period
1	0	4	0	2	0	1	0	1	0	1	0	1
2	0	4	0	2	0	1	0	2	0	3	0	2
3	0	4	0	4	0	2	0	2	0	3	0	3
4	0	4	0	4	0	4	0	4	0	4	0	4
5	1	5	1	3	1	2	1	2	1	2	1	2
6	1	5	1	3	1	2	1	3	1	4	1	3
7	1	5	1	5	1	3	1	3	1	4	1	4
8	1	5	1	5	1	5	1	5	1	5	1	5

- Transition matrix

from \ to	2	3	4
2	0.4	0.4	0.2
3	0.2	0.6	0.2
4	0.2	0.4	0.4

Equivalence of Scenario-based Model and Scenario-free Model

- The models give the same result when the scenario-based model takes all possible scenarios from the Markov process as input
- Verified numerically
- For system with 3x3 transition matrices, 16 time epochs translate to 3^16 = 43,046,721 scenarios
- Model selection
  - Scenario-based model is limited in the number of scenarios it can take
  - Scenario-free model can solve problem with the above size in 2 minutes but it is computationally challenged by the combined force of factors in the time complexity



Equity Considerations

- Ration by Schedule is considered equitable in air traffic flow management community
  - First In First Out

- Combining the previous results suggests the following implementation approach
  - Use RBS and LGF orderings
  - If RBS's solution is as good (or almost as good) as LGF's solution, then use it.
  - Else, if the gain from using LGF is big enough, use LGF's solution.

- Weighted-score priority ordering based on flight's schedule and duration